RELATIVE TIME AND FREQUENCY ALIGNMENT BETWEEN TWO LOW EARTH ORBITERS, GRACE

Willy Bertiger, Charley Dunn, Ian Harris, Gerard Kruizinga, Larry Romans, Mike Watkins, and Sien Wu

Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA

Abstract: **The two GRACE (Gravity Recovery and Climate Experiment) spacecraft were launched into a near polar circular orbit around the earth in March of 2002. The two spacecraft serve as test masses to measure the Earth's gravitational field. Both spacecraft carry ultra-stable oscillators (USO) with an Allan Deviation of a few parts in 10^{-13} for Tau = 1 to 1000 s. The USO's drive both the microwave links and GPS receivers. To cancel out long term errors on the USO's a linear combination of the 1-way microwave links is used (dual-one-way). In order to form the dual-one-way measurement and cancel our long term USO error, time must be synchronized between the two spacecraft to about 150 picoseconds. This synchronization is accomplished using the GPS data. For each spacecraft, the GPS data are used to solve for the orbital positions and the difference between the on-board clocks and a ground reference clock every 5 minutes. The relative clock is determined by the difference of these two solutions.**

Validation of the relative clock accuracy includes the solutions from overlapping data arcs which are typically less than the 150 picosecond goal and unique combination of the one-way microwave links that allows independent comparison of the GPS determine relative frequency of the USO's to a measurement made by the microwave link.

I. INTRODUCTION

Two GRACE satellites were launched on board a single ROCKOT launch vehicle on March 17, 2002, from Plesetsk (62.7° N, 40.3° E), Russia. They are in a near polar orbit at about 500 km in altitude separated by about 200 km. GRACE's primary mission is to recover both the static and time varying nature of the earth's mass distribution [*Watkins et al*., 1995; *Watkins et al*., 2000]

Fig. 1 shows the main components of the GRACE mission system. There are two GRACE spacecraft, referred to as GRACEA and GRACEB. Each spacecraft carries a codeless dual–frequency GPS receiver, a K/Ka band ranging instrument (KBR) [*Dunn et al*., 2002], an ultra-stable oscillator (USO), an accelerometer and two star trackers [*Jorgensen et al*., 1997]. The accelerometer is used to remove the non-gravitational effects from the spacecraft positions. K/Ka band measurements aided by GPS measurements of the residual effects are used to determine the gravitational forces due to the earth's mass distribution.

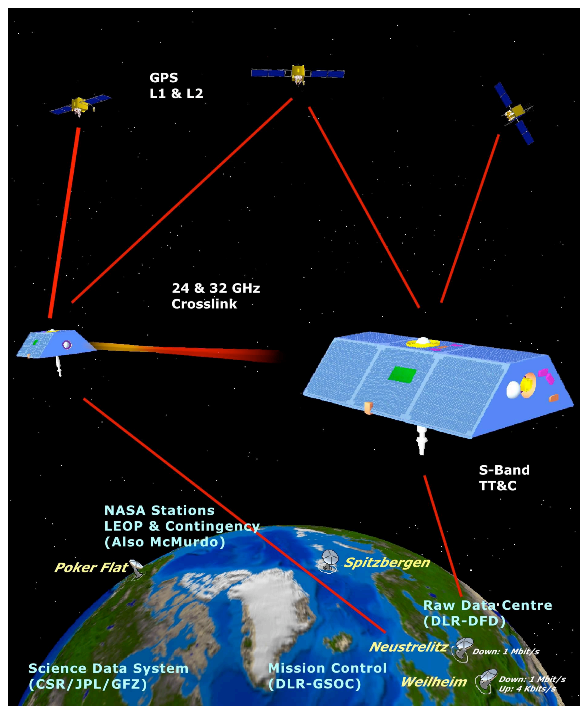

Fig. 1, GRACE System Overview

The GPS receiver and the KBR are both driven by the same USO. The KBR transmits and receives signals at K band (about 24 Ghz) and Ka band (about 32 Ghz). The four measurements of phase (2-frequencies at 2-spacecraft) are combined to measure range up to a bias in such a way that long term (longer than the light time between the two spacecraft) clock errors cancel and first order ionosphere effects are eliminated. The combination that eliminates long-term clock error is referred to as dual-one-way range [*MacArthur et al.,* 1985, *Thomas*, 1999] and can be explain briefly as follows, let

$$\phi_A = C_A(t_r) - C_B(t_t) = R + C_A^e(t_r) - C_B^e(t_t)$$

be the measurement of phase at spacecraft A, which is the difference of the clock(USO) at GRACEA at receive time and the clock at GRACEB at transmit time including any clock errors (and relativistic effects). This clock difference can further be expanded into the actual range, *R*, and a difference of clock error terms represented by the superscript e-terms above. Similarly for the phase measurement at GRACEB:

$$\phi_B = C_B(t_r) - C_A(t_t) = R + C_B^e(t_r) - C_A^e(t_t)$$

Adding these two equations together, we see that if the clock errors were constant over the light lime (difference between transmit and receive times) the errors cancel in the sum.

$$\begin{aligned}\phi_A + \phi_B = 2R &+ C_A^e(t_r) - C_A^e(t_t) \\ &+ C_B^e(t_r) - C_B^e(t_t)\end{aligned}$$

In the above argument, we are assuming near simultaneous sampling of the phase at both GRACEA and GRACEB. To achieve this near simultaneous sampling, we use GPS to align time between the two spacecraft to better than 0.15 nano-seconds (ns). Since the USO drives both the GPS receiver and the KBR instrument, precision orbit determination (POD) can be performed to determine the absolute time tag of KBR measurements and the spacecraft position [*Bertiger et al.,* 2002]. Spacecraft position is determined to about 2 cm and the absolute time is determined relative to a ground reference to less than a ns. Relative time between the two spacecraft should be better than the absolute time due to cancellation of some common mode GPS constellation errors and will be shown to be better than 150 picoseconds (ps). Tests include comparison of clock errors determined with different sets of GPS data and comparisons of the GPS determined relative clock rate relative to a linear combination of KBR phase data that reveals the relative clock rate instead of the range.

II. GPS Data Processing For Clock, Individual Clock Solutions

For an orbiting spacecraft with a GPS receiver, the process of determining the time as realized by counting the ticks of the USO driving the GPS receiver cannot be separated from the determination of the spacecrafts position. To determine the spacecraft position, a detailed set of force models are used to propagate the spacecraft position in time along with a set of stochastic accelerations to account for errors in the force models. Adjusted parameters include the initial spacecraft state (position and velocity), stochastic accelerations and a white-noise error in the clock every five minutes. See *Bertiger et al.,* 2002, for details of this solution process. Here we concentrate on the relative clock error between the two GRACE spacecraft and the validation of its accuracy.

In the GPS solution process no relativistic model of the clock behavior is included. Thus the solution for any deviation from a fixed frequency will appear in the clock solution. The solutions for the spacecraft position and clock are performed with data arcs that are 30 hours in length centered on noon of each day, thus there are six hours of common data from one arc to the next. In these six-hour overlaps, the difference in solutions gives a measure of the solution precision and accuracy. The measure of accuracy is inferred from a long history of position overlaps compared to independent measures of positional accuracy such as satellite laser ranging.

The on-board USO is used to generate the local model of the phase of the GPS signal and 1-Hz samples of this phase measurement are decimated to 5-minute samples and processed in the orbit determination and clock determination process. The GPS code measurements of absolute range are sampled every 5-minutes and smoothed against the phase measurements. USO stability is given in terms of Alan Variance, with the measured values shown in Table 1. The range of values cover a range of temperature and pressure regimes.

Table 1, USO Stability on GRACE, Pre-launch Measurement

Tau(sec)	*GRACE A ($X10^{-13}$)*	*GRACE B ($X10^{-13}$)*
0.2	11-13	13-16
2	1.3-1.4	1.7-2
10	1.2-1.3	1.3-1.8
100	1.2-1.4	1.3-1.8
1000	1.1-3.2	1.8-3.5

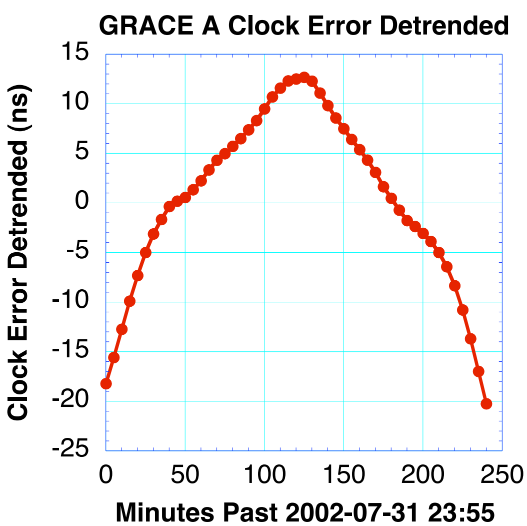

Fig. 2 GRACE A, clock error after removal of a linear trend, determined by GPS.

Figures 2 and 3 show a representative sample of the clock errors (really, both error and relativistic effects) after the removal of a linear trend. Notice the distinct difference in the two plots. The periodic nature of the plot for GRACE B is consistent with the periodic effect from general relativity with the earth as a point mass and the clock in an eccentric orbit about that mass. The change in amplitude, 2*sqrt(GM*a)e/c*, where a is the semi-major axis, e is the eccentricity of the orbit, GM is the gravitational constant times the mass of the Earth, and c is the speed of light, is consistent with the changes in eccentricity. For GRACE A, the periodic relativistic effect is dwarfed by the other errors in the clock. Of course, as noted above, the GRACE mission is only dependent on very short-term clock stability.

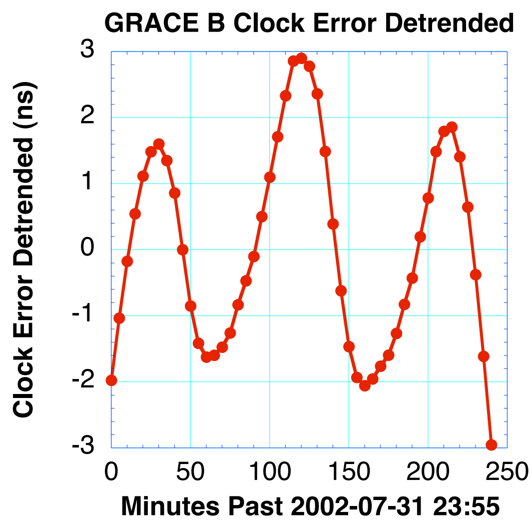

Fig. 3 GRACE B, clock error after removal of a linear trend, determined by GPS.

III Relative GPS Clock Precision/Accuracy

Since the GPS clock solutions are performed on 30-hour data arcs centered on noon of each day, we can look at the difference in the clock solutions during the 6-hour overlap period from 21:00 on one day to 03:00 on the next day. This difference is a measure of the clock solution precision and similar tests with position overlaps indicate that it is a close measure of accuracy [*Bertiger et al.,* 2002]. To eliminate edge effects in the solution process we delete an hour on each side of the overlapping period and look at differences from 22:00 to 02:00 on the next day. For relative clock precision and accuracy, we look at the difference of GRACE A clock in the overlap – the difference in GRACE B clock in the overlap. This difference removes any effects of the reference clock, since the reference clock is common to both GRACE spacecraft but may switch from 30-hour arc to 30-hour arc. Fig. 4 shows a histogram of the RMS of the overlap differences for almost one year, from April 1, 2002 to March 16, 2003. The median RMS overlap is 68 ps, well within the bounds of the 150 ps mission requirements.

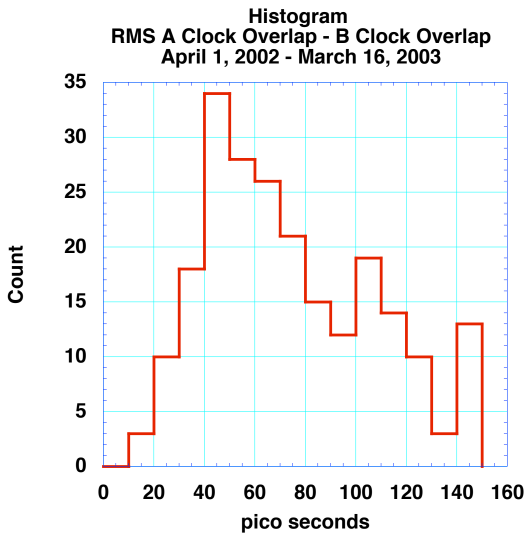

Figure 4, Histogram of Clock Overlaps, Median: 68 ps

IV Clock Rate KBR Compared to GPS Measurement

As a final validation of the GPS clock solution, we can use the KBR data itself as an independent measure of the clock rate (frequency) since both data streams are driven by the same set of USO's. A few equations are necessary to explain the comparison and the relevant combination of KBR data.

Let

$\phi_2^1(t) =$ K or Ka phase up to a bias measured at spacecraft 1 (we switch to using 1 and 2 for A and B in the equations) at true time t

(1) $\phi_2^1(t) = \phi^1(t) - \phi^2(t - \tau_2^1) + I$

where, $\phi^1(t)$ is the phase generated at true receive time t at spacecraft 1 and τ_2^1 is the travel time from spacecraft 2 to 1 including light time and other delays, I is an ionosphere induced phase shift , and $\phi^2(t - \tau_2^1)$ is the phase generated at spacecraft 2 at true time $t - \tau_2^1$.

Local time, at true-time t, at the receiver, is defined by $\bar{t}^i(t) = \phi^i(t)/f_i$ where f_i is the assumed nominal *rf* frequency for spacecraft i, (i = 1, 2) at K or KA band. An arbitrary constant, synchronizing the epochs of true time and local time is omitted.

The corresponding equation for the phase measured at spacecraft 2 is obtained by just interchanging the numbers 1 and 2 in equation for the measured phase at spacecraft 1.

Taking the difference of the measured phases at spacecrafts 1 and 2 and dividing by the sum of the frequencies at the two spacecraft we obtain after dropping the ionospheric phase shift for now:

(2)
$$\frac{\phi_2^1(t) - \phi_1^2(t)}{f_1 + f_2} = \frac{f_1 \bar{t}^1(t)}{f_1 + f_2} - \frac{f_2 \bar{t}^2(t)}{f_1 + f_2} + \frac{f_1 \bar{t}^1(t - \tau_1^2)}{f_1 + f_2} - \frac{f_2 \bar{t}^2(t - \tau_2^1)}{f_1 + f_2}$$

Writing local time, $\bar{t}$, as $t - \delta(t)$, and differentiating equation (2) with respect to true time t,

(3)
$$\frac{\dot\phi_2^1(t) - \dot\phi_1^2(t)}{f_1 + f_2} = \frac{2 f_1}{f_1 + f_2} - \frac{2 f_2}{f_1 + f_2} + \frac{-f_1 \dot\tau_1^2}{f_1 + f_2} - \frac{-f_2 \dot\tau_2^1}{f_1 + f_2} + \frac{-f_1 \dot\delta^1(t)}{f_1 + f_2} + \frac{f_2 \dot\delta^2(t)}{f_1 + f_2} + \frac{-f_1(1 - \dot\tau_1^2)\dot\delta^1(t - \tau_1^2)}{f_1 + f_2} + \frac{f_2(1 - \dot\tau_2^1)\dot\delta^2(t - \tau_2^1)}{f_1 + f_2}$$

The time delay between reception and transmission, $\square$, is typically less than a millisecond with the spacecraft separation of about 200 km and the second derivative of the clock error is typically less than 10^{-14} s/s^2(bounds on this are easily computed from the GPS solution). Thus $\square(t) = \square(t \square \square)$ to about 10^{-17} s/s and $\square$ can be dropped from the function argument in the last two terms. The argument that $\square$ has a small effect on the value of the rate of change of time can be applied to show that even though we have implicitly assumed in (3) that the data are sampled at each spacecraft at the same true time *t*, this simultaneous sampling need only be good to the millisecond level to compare rates to the 10^{-17} s/s level. Let $\square f = f_2 \square f_1$ be the difference in the *rf* frequencies at the two spacecraft, about 500 Khz, and $\square$, the mean derivative of the travel time between the two spacecraft (the derivative of the travel time differs between the two spacecraft by at most $2\text{x}10^{-13}$ from March 2-7, 2003, so we can substitute the mean of the two values in the last two terms),

(4)

$$\frac{\square_2^1(t) \square \square_1^2(t)}{f_1 + f_2} \square \frac{\square 2 \square f}{f_1 + f_2} + \frac{\square f_1 \square_1^2}{f_1 + f_2} \square \frac{\square f_2 \square_2^1}{f_1 + f_2} \square \frac{\left[\square^1(t) \square \square^2(t)\right]}{2} + \frac{\left[\square^1(t) \square \square^2(t)\right]}{2}(1 \square \square) + \frac{\square f\left[\square^1(t) + \square^2(t)\right]}{2(f_1 + f_2)} + \frac{\square f\left[\square^1(t) + \square^2(t)\right]}{2(f_1 + f_2)}(1 \square \square)$$

Finally solving for the difference in clock rate,

(5)

$$\left[\square^1(t) \square \square^2(t)\right] \square \frac{1}{(1 \square \square/2)} \left\{ \frac{\square_1^2(t) \square \square_2^1(t)}{f_1 + f_2} \square \frac{\square 2 \square f}{f_1 + f_2} + \frac{f_2 \square_2^1 \square f_1 \square_1^2}{f_1 + f_2} \right\} + \frac{\square f\left[\square^1(t) + \square^2(t)\right]}{f_1 + f_2}$$

The second term inside the parenthesis on the right hand side is a large known bias due to the frequency offset between the spacecraft. The $\square$ terms are easily computed to high accuracy with the GPS position solutions. The last term of equation (5), will yield a value that is close to constant. For March 2-7, 2003, the maximum deviation of the sum of the last term from its mean on each day was less than $1.8\text{x}10^{-16}$. Also note that all the terms in equation (4) are the same for both the K and Ka frequencies since they either are independent of the nominal frequency or have a ratio of frequency (K = (3/4)KA for each spacecraft). Since the ionosphere free combination exactly sums to 1, all the arguments for the terms on the right hand side are unaffected by eliminating the small differential ionosphere term that was dropped in forming the difference in equation (2).

Figure 5 shows the relative clock rates as determined by GPS and KBR on a typical day, March 2, 2003. At the scale of the drift in rate, there is almost no difference in the two measures of relative clock rate. Figure 6, displays the difference in the two determinations of relative clock rate. There is an overall mean difference of –0.065 ps/s which we cannot currently explain. This rate difference would mean clock difference of 5.6 ns in a day, too large for an error in GPS system. The RMS about the mean is 0.059 ps/s. The periodic variation in Fig. 6 is consistent with periodic errors in position, 0.06 ps/s corresponds to about 1.8 microns/s (0.06*speed of light), about the magnitude of rate of orbital along track positional errors. Orbit errors, since they are dynamic in nature, tend to have periods the same as the orbital period, about 90 minutes. Thus the periodic errors are probably due to GPS clock rate determination errors and could be reduced if the clock errors were treated as some constrained correlated process noise instead of unconstrained white-noise.

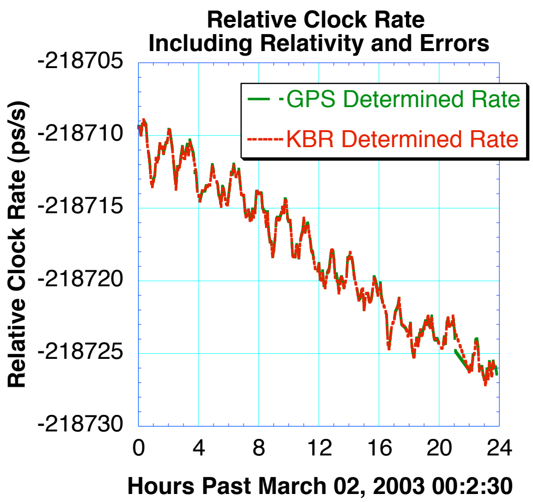

Figure 5, Relative Clock Rates as determined by GPS processing and KBR measurement.

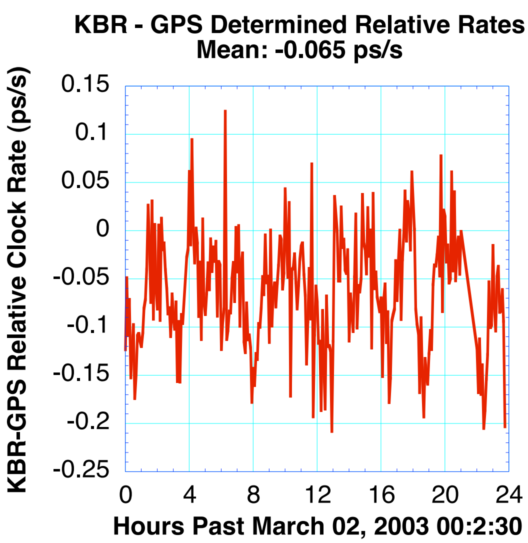

Figure 6, Difference of KBR and GPS determined clock rates (difference of curves in Fig. 5)

V CONCLUSIONS

The relative clock between the two orbiting GRACE spacecraft can be determined to better than 150 ps. A new method of determining relative clock rate using inter-satellite phase measurements gives agreement to the GPS determined values consistent with errors in the GPS system. GPS clock determination could be improved by taking further advantage of the clock stability.

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